CAPACITORS CAN RADIATE - SOME CONSEQUENCES OF THE TWO-CAPACITOR PROBLEM WITH RADIATION

T.C. CHOY

Abstract. We fill a gap in the arguments of Boykin et al [1] by not invoking an electric current loop (i.e. magnetic dipole model) to account for the radiation energy loss, since an obvious corollary of their results is that the capacitors should radiate directly even if the connecting wires are shrunk to zero length. That this is so is shown here by a direct derivation of capacitor radiation using an oscillating electric dipole radiator model for the capacitors as well as the alternative less widely known magnetic 'charge' current loop representation for an electric dipole [2]. Implications for Electromagnetic Compliance (EMC) issues as well as novel antenna designs further motivate the purpose of this paper.

1. Introduction

The recent paper by Boykin et al [1] on the two-capacitor problem with radiation has a gap in their arguments which we shall fill in this short note, which also serves to provide an alternative and perhaps more direct derivation of their key results. An important corollary of their results which we shall show explicitly here implies that there can be radiation *from the capacitors*. This radiation source is often not discussed in many standard texts on Electromagnetism [4, 5, 6] and in particular Antenna theory [7, 8, 9] and others, with the exception perhaps of [2] who highlighted the duality of magnetic dipole radiation due to a physical current loop and the electric dipole radiation due to a fictitious magnetic charge current loop in his treatment. His seminal biconical antenna [2] is perhaps one of the few exactly solvable antenna models that also demonstrates capacitor radiation. The importance of direct radiation from capacitors has important implications for Electromagnetic Compatibility (EMC) issues [3] as well as alternative non-conventional antenna designs currently hotly debated in the engineering and amateur radio literatures [13, 14]. It is hoped that this note shall help to clarify the basic physics involved in these alternative antennas.

We shall first discuss a corollary of the results of Boykin et al [1]. There is no doubt that their derivation demonstrates that the missing energy can be accounted for by radiation as they have shown using a magnetic dipole, current loop model. We can then ask the question as to what would happen if the current loop is shrunk to an infinitely small radius. Their result then indicates that the capacitors must now become the source of radiation. This can be seen in two ways. 1) In their derivation eqn(16) indicates that the physical dimension of the current loop b (which is absorbed in the constant K) eventually cancels out so that in fact the limit $b \to 0$ can be taken in eqn(16) without mathematical difficulties. 2) Owing to an equivalence theorem between a circular and a square loop radiator (see their Fig 1) of identical area [9], we can now shrink the wire dipole radiators to zero length, from which energy conservation would imply that the capacitors must themselves serve as radiators. That this is so will be demonstrated in the next two sections.

2. Oscillating electric dipole model

. In the long wavelength limit, the two capacitors connected in parallel with zero length wires can be viewed as an oscillating electric dipole due to a series capacitor of value C_s, (see Fig 3 of [1]) . The power intensity of radiation from such a dipole of moment p is now given by [11]:

$$P_{rad} = \frac{1}{6\pi\epsilon_0 c^3}\ddot{p}^2. \tag{1}$$

In our ideal capacitor model considered as two parallel plates with separation ℓ, then:

$$\ddot{p} = C_s \ddot{V_c}^2 \ell^2. \tag{2}$$

Since now:

$$V_X = \frac{P_{rad}}{I} = \frac{P_{rad}}{\dot{Q}} = \frac{P_{rad}}{\dot{V}_c C_s}, \tag{3}$$

the non-linear differential equation eqn(12) of [1] immediately follows in a similar way using their lump circuit model to account for the radiation resistance X i.e.:

$$\ddot{V}_c^2 + \frac{1}{K_C C_s}\dot{V}_c V_c = 0. \tag{4}$$

except that now we have:

$$K_C = \frac{\ell^2}{6\pi\epsilon_0 c^3}. \tag{5}$$

The rest of the proof for the radiation energy follows identically as [1] so that we do not need to reproduce them here. This result can also be derived using an analogous model as [1], but this time using a fictitious magnetic current loop model [2]. As these formulas are not often used in standard texts we shall provide the details in the next section, highlighting the advantage that this model will be more useful in terms of evaluating actual antenna radiation characteristics in more realistic capacitor antennas using standard formulas. Before we do so it is perhaps worth drawing to attention that the point dipole model is an extreme limit for the capacitor since like the corresponding short current dipole for wires [4, 5, 6, 7, 8, 9], and as for long wires [9] of order λ, the contributions should be added vectorially for each element and integrated over the capacitance area for a λ size capacitor.

3. Magnetic current loop model

The magnetic current loop model uses the fact that Faraday's law of magnetic induction can be used to define a magnetic 'charge' current as the source for an electric dipole field. In this case we shall have a not frequently used vector potential **F** such that:

$$curl\ \mathbf{F} = -\mathbf{D}, \quad \mathbf{H} = -\frac{\partial \mathbf{F}}{\partial t}; \tag{6}$$

and thus also:

$$\mathbf{F} = \frac{\epsilon_0}{4\pi}\int \frac{I_M(t - r/c)}{r} dl, \tag{7}$$

where I_M specifies the magnetic current of the loop [10]. Once again the results of [1] applies by analogy in particular their eqn(8), upon replacing ϵ_0 by μ_0:

$$P_{rad} = \frac{\pi b_m^4}{6\mu_0 c^5}[\ddot{I}_M(t - r/c)]^2, \tag{8}$$

where b_m is now the magnetic current loop radius which should be at least the radius of the assumed circular parallel plate capacitors. In view of Faraday's law $V_c = -I_M$:

$$V_X = \frac{P_{rad}}{I} = K_M \frac{\ddot{V}_c^2}{C_s \dot{V}_c}, \tag{9}$$

and once again eqn(4) follows except that now K_M is defined as:

$$K_M = \frac{\pi b_m^4}{6\mu_0 c^5}, \tag{10}$$

which as we can see is a less efficient radiator. For the same size loop $b_m = b$ the ratio:

$$\frac{K}{K_M} \approx 10^5. \tag{11}$$

The implications of this result for EMC is also important. Since in terms of spectral content, a less efficient radiator would in the capacitor system tend to spread the energy over a wider spectrum, i.e. assuming ideal capacitors with no internal losses. Both magnetic or electric dipoles have an intensity that goes as a fourth power of frequency ω^4 [11]. Indeed as noted in [1] the radiation resistance is given by $R_{rad} = Ks^4$ in the frequency domain. Hence for a smaller K the radiation time constant $\tau = R_{rad}C_s$ is smaller which implies a wider spread in radiation energy. Note however that the point dipole model of the previous section yields a different picture. Here the efficiency factor is given by:

$$\frac{K}{K_C} = \frac{2\pi^2}{c^2}\frac{b^4}{\ell^2} \approx 10^{-15}\frac{b^4}{\ell^2} >\approx 10^7 C_s^2. \tag{12}$$

which for large capacitances can be comparable to the wire loop. Practical capacitance antennas, depending on frequencies will behave somewhere between a point electric dipole versus a magnetic current loop, as we shall see in the next section [12].

4. Capacitance antennas

In recent years there has been controversies in the engineering community regarding certain capacitance antennas, patented in the US and Britain [13, 14], which purports to use Poynting vector synthesis for its operational principles. These are in fact now commercial products that have received contradicting support from broadcasting applications. We do not however subscribe to the theory of Poynting vector synthesis [13, 14]. Nevertheless, the analysis from the last section shows that (a) capacitance radiation is a reality and (b) their efficiencies based on the idealized magnetic 'charge' current loop model which is close to their practical counterparts, depend very much on the capacitance disc sizes. To achieve similar efficiency as a wire loop, the capacitance antenna loop b_m needs to be an order of magnitude bigger than the magnetic loop area b at low frequencies:

$$b_m = 10b. \tag{13}$$

It would seem that any low profile advantages achieved in the use of capacitance antennas has to be paid for by much larger capacitance disc areas. However at high frequencies in which the point electric dipole model is approached, capacitors might be better radiators, which might in fact find useful applications for the new digital spread spectrum modes of transmission. Having said this, most antennas are not meant to act as broadband radiators. Wire antennas resonate at the operating frequency by making use of either free space capacitances leading to the empirical formula [7, 8, 9]:

$$L = \frac{143}{f_{Mhz}}, \tag{14}$$

for the length of a half-wave dipole antenna in metres up to several hundred MHz. In the same way capacitance antennas can resonate using free space inductances. The corresponding formula for the capacitance dipole, including its relative performance would be an interesting exercise for the student. For practical systems a lump circuit analysis [1] by introducing an inductance L which could be both a sum of stray plus external inductances would suffice. However modern Pspice software and various antenna modelling software do not include the radiation resistance from capacitances discussed here, so some care needs to be exercised in their use.

5. Conclusion

We have filled a gap in the discussion of the radiation from the transient switching of charges between two capacitors. We showed that without a wire loop, or in the limit where the wire lengths are infinitesimal, the capacitance system will radiate, using a point electric dipole model or more appropriately a magnetic 'charge' current loop model. These results should be added to modern texts on electromagnetism and antenna theory. The implications of our results for EMC directives and for novel antenna designs are significant and should also be noted in physics and engineering teaching courses.

References

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10. The SI units for magnetic 'charge' current is in fact in volts.
11. L. D. Landau and E. M. Lifshitz, "The Classical Theory of Fields " Vol 2 in Course of Theoretical Physics translated from Russian, Pergamon Press, Oxford (1975).
12. This property is of course similar to wire antennas.
13. U.S. Patent no 6486846 B1 filed by T. Hart, see also "The EH Antenna - Exceptional or Hype" by B. Prudhomme in "Monitoring Times" **22** (4) 22 (2003), see also http://www.eh-antenna.com.

14. British Patent no 9718311 and U.S. Patent no 6025813 filed by M. Hately and F.Kabbary, see also "Poynting Vector Synthesis and the CFL" by P. Hawker in RadCom - The Radio Society of Great Britain members magazine **78** (8) 63 (2002) and references quoted there in.

Centre for Materials Research University College London Gower Street London WC1E 6BT